Continuous Random Variables

A random variable X is said to have a continuous distribution if there exists a non-negative function f s.t. $P(a < X \leq b) = \int_a^b f(x)dx$, for all $-\infty \leq a < b \leq \infty$.

1. Probability Density Functions

We call f a probability density function (PDF), the density curve, or the density of X if it is nonnegative, i.e. $f(x) \geq 0$ for all x and the total area under the PDF is 1, i.e. $\int_{-\infty}^{\infty} f(x)dx = P(-\infty < X \leq \infty) = 1$.

1.1. Remark: the PDF $f(x)$ itself is not a probability, it is the area underneath that represents a probability.

1.2. Remark: for continuous random variables X, $P(X = x) = \int_x^x f(u)du = 0$.

1.3. Remark: the PDF of a continuous random variable need not be continuous.

2. Cumulative Distribution Functions

For any random variable X, whether it is discrete or continuous, its CDF is the function defined by $F(x) = F_X(X) = P(X \leq x)$. One can get the CDF of a continuous random variable by integrating its PDF.

2.4. Remark: a CDF is non-decreasing

2.5. Remark: a CDF is bound by 0 and 1

2.6. Remark: the CDF of a continuous random variable must also be continuous

2.7. Remark: finding area under an interval in PDF $=$ finding difference in values in endpoints in CDF

So for any number a, $P(X > a) = 1 - F(a)$ and for any two numbers a, b, $P(a \leq X \leq b) = F(b) - F(a)$.

3. CDFs for Discrete Random Variables

We can similarly define a CDF for a discrete random variable by using a piecewise function.